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ORIGINAL PAPER

Numerical Modeling of Fluid Sand Boiling from Coastal Dike Foundations

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Abstract In the present study, by simulating an experimental model of a coastal dike (8 m long, 1 m deep and 1 m wide) by FLAC-2D software based on finite difference method, the soil failure mechanism caused by seepage force has been performed. To solve this problem in order to study the boiling, the soil foundation should be analyzed at the most critical condition. The numerical model presented in FLAC-2D is properly able to simulate the flow properties (flow velocity, hydraulic conductivity and seepage discharge). Comparing the results from the numerical model with experimental data shows that the numerical model well predict soil behavior including boiling and soil displacements based on stress–strain analysis. The simulation indicated that dilation and internal friction angles also influenced the stability coefficient.

Keywords Sandy soils · Boiling · Hydraulic structures · Failure · Effective stress

1 Introduction

One of the destructive factors of coastal dikes, soil barriers, concrete barriers and other hydraulic structures built along coastal areas is seepage flow through the foundation. However, in many cases it is difficult to assess and foresee the effects of seepage through the body and/or foundations of such structures. As a general rule, the quantity of seepage force and seeping water through the body and foundation of hydraulic structures should be minimized so that the stability of the structures is safe guarded. Coastal dikes are common examples of hydraulic structures built along the coastal areas and founded on deltaic/alluvial materials. Due to the nature of foundation materials (loose, clean sand soils), these structures are subjected to liquefaction/boiling when transient hydraulic gradients are generated (Sedghi-Asl et al. 2012).

Allowable range for optimal design of hydraulic structures against destructive phenomena such as boiling or piping can be detected (Kuriqi and Ardiclioglu 2005). In addition, flood control systems are used for systems such as dikes, pumping stations and

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drainage systems and management of the destructive phenomenon of boiling in these systems is very important (Teng and Chen 2013). If flood control does not work correctly due to boiling, seepage and instability, it can not be controlled and then would be a catastrophic phenomenon (Kuriqi et al. 2016). Sedghi-Asl et al. (2015) investigated internal erosion under a spillway rested on Shah-Ghasem embankment dam, in southwest of Iran. They concluded that the foundation material (Marl stone) was the main reason of occurring internal erosion.

The first studies on minimizing seepage flow through earth dams have been conducted by Tarzaghi (1946). He analyzed the seepage flow through body and foundations of earth dams considering the coefficient of permeability of the materials at different sections of dam. Based on his findings, the less the core permeability compared to shell and foundation, the more would be the reduction in seepage quantity. He also analyzed the seepage quantity from shell and foundation considering different penetration depths of cut-offs. His early works were based on seepage analysis using flownet for each case (Rahimi 2003). During the past decades many other researchers have reviewed the seepage issues analytically and numerically. In most such researches, the agreement between of results of the numerical model and experimental data were evaluated (Sedghi-Asl et al. 2005, 2012).

Boiling is upward movement of soil grains when a small prism of soil at downstream of a cut off wall or sheet pile, doesn't have enough resistance to neutralize the uplift pressure or upward seepage force. Terzaghi (1943) defined the critical hydraulic gradient (sinking slope, i_c) as a parameter to control boiling of materials. McNamee (1949), defined the safety factor against boiling as the ratio of the critical hydraulic gradient to the exit hydraulic gradient according to Eq. 1,

$$F = i_c / i_e \tag{1}$$

Lane (1935) defined the flow creep length from deviating foundation equal to the total vertical lengths plus one-third of horizontal length of seepage path. He suggested an experimental index, c, for different types of foundation materials to properly control boiling and internal erosion (piping). Lane defined the weighted creeping factor as ($c = L/\Delta h$), where L is the creep length and Δh is the difference between upstream and downstream water levels and c is Lane's weighted index. If the calculated weighted index for a structure founded on piping sandy soil is higher than the value suggested by Lane, then it would be safe against boiling and internal erosion.

Neuman and Witherspoon (1970) simulated the seepage flow through earthen dam by finite element method which is mathematically more complicated. Javan and Farjood (1993) suggested a computer model that was able to estimate the uplift pressure in hydraulic structures using different approaches. They compared their results with piezometric data from Doroodzan Dam (located in Fars Province of Iran) and found that there is only a 4 percent difference between the results of computer modeling and field data.

Sedghi-Asl et al. (2005) reviewed the effect of optimal position of the sheet pile on seepage and flow velocity in hydraulic substructures by using a numerical model and found that the best location for optimal control of seepage and boiling is at heel and toe of the structure.

Benmebarek et al. (2005) studied numerically the destruction caused by seepage at lower part of a cut-off installed in sandy foundation. They used FLAC-2D model which is based on explicit finite difference to analyze the boiling caused by seepage flow. The main objective of their work was to determine the conditions when boiling of soil grains may cause failure. Findings of this work have not been compared with experimental data.

Sedghi-Asl et al. (2012) studied the effect of different parameters on seepage flow through the foundation of coastal dikes. They built an experimental flume 9 m long, with 1 m width and height using steel framing and glass and Plexiglas walls. Fine, clean sand was used as the foundation materials. Their results showed that for minimizing seepage rate and control of internal erosion, the ratio of blanket length (L_B) to the upstream water depth (H) and the sheet pile depth (d) to foundation thickness (D) in order to minimize the seepage and control the internal erosion, are 8 and 0.8, respectively.

The critical hydraulic head differences for possible seepage failure were analyzed using prismatic failure concept by Tanaka & Verruijt (1999). Tanaka & Yokoyama (2006) investigated the effect of supplementary injection of jet grouting under sheet piles for single and double-sheet-pile-wall and circular cutoff-walls. They concluded that the critical hydraulic head differences between upstream and downstream goes up with increasing the injected length of jet grouting.

Recently Tanaka et al. (2012) investigated seepage failure of bottom soil within a double-sheet-pile wall for a case study. They studied seepage and boiling by means of finite element method (FEM) and determined safety factor against seepage failure.

Review of the previous studies shows that they have mainly been based on mathematical and numerical analysis of seepage flow and less comparison has been made with experimental data. Despite extensive research on seepage and boiling, this phenomenon is still not predicted accurately, safe and reliably before the construction of hydraulic structures. Most of the numerical models used in general and based on the pore pressure of this phenomenon are evaluated. Performing laboratory models are often time consuming and non-economic. Thus, the results of numerical analysis alone may not provide an accurate and reliable assessment of stability of the hydraulic structures founded on boiling susceptible materials. In the present study, numerical modeling by FLAC-2D was employed for an experimental work, where seepage path, and boiling was analyzed using the experimental data of Sedghi-Asl et al. (2012).

2 Modeling

Modeling in FLAC-2D has been done according to the experimental model that This model has been made in Water Research Central Laboratory of Irrigation and Reclamation Department of Tehran University (Sedghi-Asl et al. 2010, 2012). Table 1 shows the geometry of the model. Table 2 states the sheet pile position.

We know, blanket length = L_B, water head at upstream = H, and depth of foundations = D. Table 3 lists all the desired scenarios.

Figure 1 shows the the experimental model. Figure 2 shows the schematic diagram of the experimental model.

Table 2 Properties of the sheet pile used in this study

	Material	Location
Properties of the sheet pile	Plexiglass	5 m from the start

The analysis was carried out by the computer code of FLAC-2D program which is an explicit finite difference based on Lagrangian analysis. In this program, an explicit time-dependent approach was used to solve the algebraic equations. It should be noted that contrasting the public belief, finite difference approach isn't limited to rectangle zones. In FLAC-2D, based on Wilkins' studies, a method wad used by which it is possible to calculate differential equations based on finite difference method from zones with different forms (Wilkins 1964). FLAC-2D usage mechanism includes: 1- creating a finite difference grid; 2- applying the treatment model and material properties; and 3- applying boundary conditions.

Governing Equations in this study is that, in FLAC, a gradual formulation of flow transformation and fluid propagation processes is carried out simultaneously, based on the linear theory of the Biot static network. The various numeric rules in FLAC are:

2.1 Transport Law

The fluid transport is expressed by Darcy's law

$$q_i = -k_{ij}^{a} \frac{\partial}{\partial x_j}\left(p - \rho_w g_k x_k\right) \tag{2}$$

where q_i is the specific discharge vector, and $-k_{ij}^{a}$ is the apparent mobility coefficient, which is a function of the saturation s.

2.2 Balance Laws

Fluid mass balance equation is

Table 1 Labortaory model properties and geometry

	Length (m)	Wide (m)	High (m)	Material	Foundation depth (m)	Foundation soil type
Laboratory model properties and geometry	8.4	0.95	1	Steel	0.5	Fine clean sand

Table 3 Properties of the sheet pile

	D (m)	L_B (m)	H (m)
Different scenarios	0.1, 0.2, 0.3, 0.4	1, 2, 3, 3.5	0.05, 0.1, 0.15, 0.2, 0.25

With zero head at downstream

Fig. 1 A general view of the experimental model

$$\frac{\partial \xi}{\partial t} = -\frac{\partial q_i}{\partial x_i} + q_v \tag{3}$$

where ξ is the variation of fluid content, and q_v is the volumetric fluid source intensity. The balance of momentum has the form

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = \rho \frac{\partial \dot{u}_i}{dt} \tag{4}$$

where $\rho = (1 - n)\rho_s + n\rho_w$ is the bulk density; ρ_w, ρ_s are the densities of the solid and fluid phase, respectively. Note that $(1 - n)\rho_s$ corresponds to the dry density of the matrix,$\rho_d\psi$ (i.e., $\rho = \rho_d + n\rho_w$).

2.3 Constitutive Laws

The response equation for the fluid inside the pores depends on the saturated volume. Absolutely saturated, $s = 1$, $k^a_{ij}(s) = k_{ij}$ and the fluid can sustain a tension up to a limit, ρ_d, as described in Sect. 1.5.6. For s = 1, the response equation is:

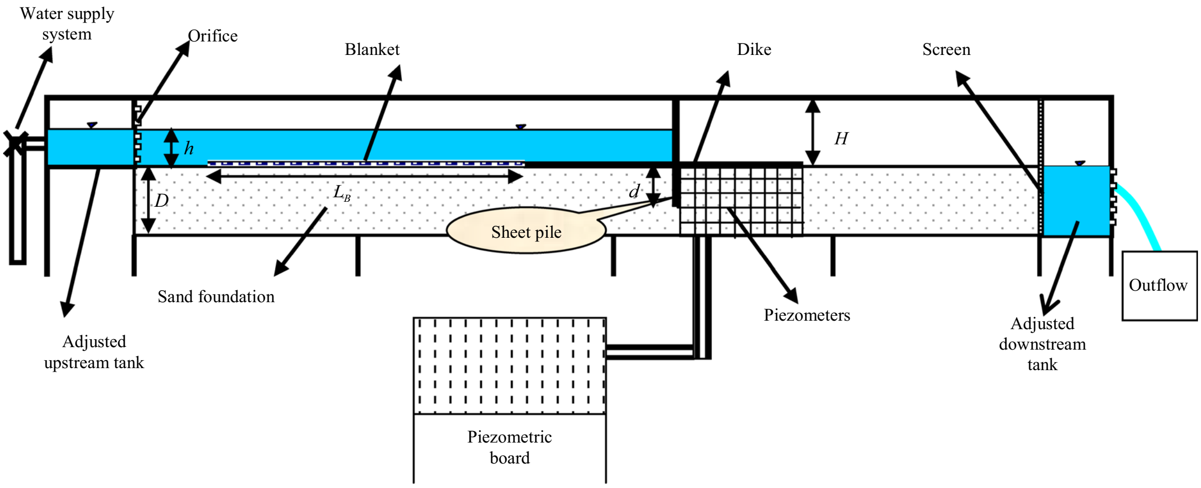


Fig. 2 Schematic diagram of laboratory flume (Sedghi-Asl et al. 2010)

$$\frac{\partial p}{\partial t} = M\left(\frac{\partial \xi}{\partial t} - \alpha \frac{\partial \in}{\partial t}\right) \tag{5}$$

where M is Biot modulus, α is Biot coefficient and $\in$ is the volumetric strain. In FLAC, the compressibility of grains is neglected compared to that of the drained bulk material, and we have:

$$M = \frac{k_w}{n} \tag{6}$$

$$\alpha = 1 \tag{7}$$

One of the techniques for specific applications in FLAC is adoption of Tarzaghi and Taylor's equation. Three forces are applied on the soil volume unit in rigidity matrix that includes: soil weight, buoyancy and force resulted from seepage, while FLAC-2D also operates based on the same rules (Bear 1972). In this program, the forces have been considered automatically and the governing equation (normal consolidation) is as following (Bear 1972):

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho_s g_j = 0 \tag{8}$$

where ρ_s is non-drained density, g_j is gravitational vector and σ_{ij} is effective stress. Undrained density may be expressed in terms of drained density, ρ_d, and fluid density, ρ_w, using the expression

$$\rho_s = \rho_d + ns\rho_w \tag{9}$$

where n is porosity, and s is saturation. The definition of effective stress is

$$\sigma_{ij} = \sigma'_{ij} - p\delta_{ij} \tag{10}$$

Substitution of Eqs. (9) and (10) in Eq. (8) gives, after some manipulations,

$$\frac{\partial \sigma'_{ij}}{\partial x_j} + \rho_d g_i - (1-n)\frac{\partial p}{\partial x_j} n\gamma_w \frac{\partial \phi}{\partial x_j} = 0 \tag{11}$$

where we have introduced fluid unit weight, γ_w, and piezometric head, ϕ, as

$$\gamma_W = \rho_W g \tag{12}$$

$$\phi = \frac{P}{\rho_W G} + \frac{x_k g_k}{g} \tag{13}$$

Note that g is the gravitational magnitude, ϕ,n are piezometric height and porosity, respectively, $\rho_d g_i$ is soil weight, $(1-n)\frac{\partial p}{\partial x_j}$ is floating force and $n\gamma_w \frac{\partial \phi}{\partial x_j}$ shows the seepage force (FLAC-2D 2000).

The summary of the modeling process was as follows:

1. Modeling the geometry of the experimental model.
2. Designing a behavioral model for soil.
3. Modeling the flow of water in the soil.
4. Modeling of soil mechanical behavior.

This is the flowchart of modeling in FLAC (Fig. 3).

Experimental model of the coastal dike has dimensions of 1.8 m long, 1 m high and 0.95 m wide, 0.5 m from the height of the dike with fine sand and clean. The geometry of the model conforms to the figure in FLAC in two dimensions, conforming to the dimensions of the laboratory model. This model shows 0.5 sandy sand cast in flume. Mesh generating was performed according to Fig. 4. It was not defined for the modeling of the element flume walls.

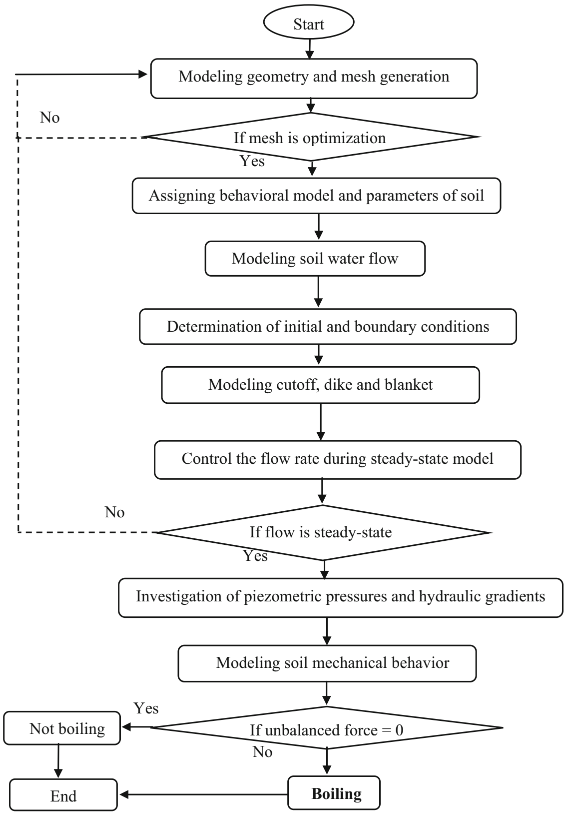


Fig. 3 Modeling steps in FLAC

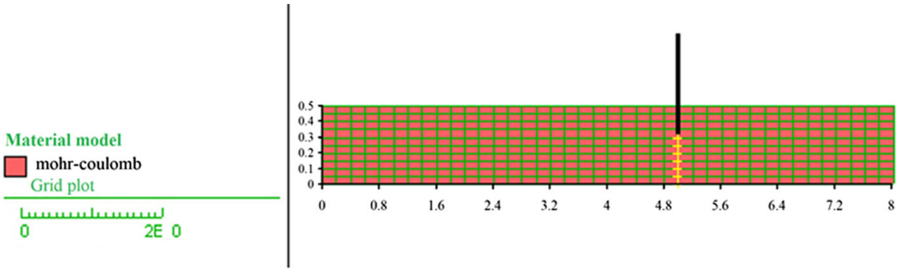


Fig. 4 Numerical model grid in numerical setting

The boundary conditions were exactly the same as the laboratory model of boundary conditions.

The left and right sides of the model are completely fixed in the direction of X and the floor of the model is defined in the direction of Y.

As a boundary condition, it was defined as a fitted support. Different ways were tested in FLAC to model the sheet pile. On the top of this sheet pile location, the beam element was made of glass plexiglass. For easy modeling of dike and blanket, the laboratory model has made from plexiglass. In the numerical model, the same characteristics of plexiglass were exactly allocated.

The boundary conditions are such that at the top of the sheet pile, it is possible to enter the water and at the bottom there is a possibility of water exiting. Tank water entry and exit are set at 25 cm upstream and downstream water levels on the surface of the foundation. In addition, downstream of right is a tower that prevents the passage of sand, but the water passes easily and filling the reservoir that is thereafter up to 0.5 m high, causing The environment remains saturated.

In the next step, considering the fact that sheet pile and foundation materials are different, modeling the interface between the wall and soil should be modeled. In other words, the friction force between sand and sheet pile should be considered as an important factor in analysis. In this research, the material properties and stiffness of each section are separately specified. In Fig. 5, by using Coulomb Rule, the interface between the wall and soil has been simplified. To apply the characteristic of the interface of the two elements, the vertical stiffness k_n and shear stiffness k_s should be calculated for either side. For carefully modeling, the friction angle between Plexiglas sheet and **sandy foundation is required.

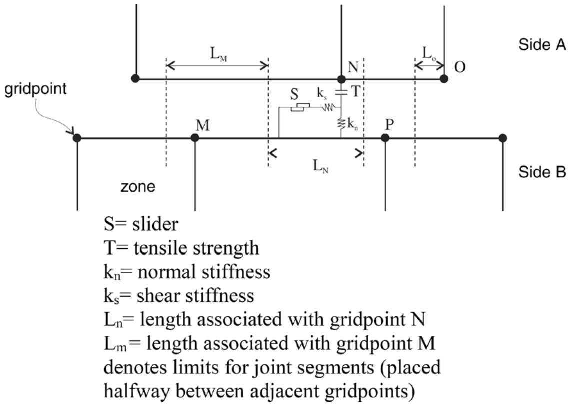


Fig. 5 Modeling the two different element connections in FLAC-2D (2000)

The most critical friction angel for this study was used as 22° (Barchard 2002). Equivalent stiffness or modulus of elasticity is defined as

$$E_S = \left[\frac{(K + \frac{4}{3}G)}{\Delta Z_{\min}}\right] \tag{14}$$

In this equation, E_S: Equivalent stiffness, K: Bulk modulus,G: Shear modulus, $\Delta Z_{\min}$: The minimum size of element. Finally, values of shear stiffness and vertical stiffness are equal to:

$$k_n = 10E_S \tag{15}$$

$$k_S = 10E_S \tag{16}$$

The equivalent stiffness introduces the stiffness of the most rigid zone in the studying border (FLAC-2D 2000). Furthermore, for modeling the interface between sheet pile and sand, the friction angle between Plexiglas screen and sand should be determined. By conducting the direct shear test, this angle was determined as 26° and 22° as the most critical case.

The Mohr–Coulomb's model has been used in this modeling. The following properties were considered for the foundation fine sand; Density = 1350 kg/m^3, Modulus of Elasticity, E = 10–25 Mpa, Poisson Ratio ν = 0.25 (Barchard 2002, Das 1941, Subramanian 2008). Considering the most critical case, for very fine sand, it is assumed E = 10 Mpa.

In the next step, the fluid flow and mechanical changes of the porous medium are simultaneously considered. According to experimental settings, boundary conditions have been applied in a way that upstream heads were at 5 levels of 0.5, 1, 1.5, 2, 2.5 m, with zero head downstream.

Bottom and left-hand faces of the foundation are completely impermeable. At the right-hand face of the experimental model, water flow can pass through a fine–mesh. During course of experiments, the upstream and downstream heads were kept constant and thus, the flow is in the steady-state. The highest potential for creation of boiling in the model is at upstream head of 25 cm.

In the experimental model, at the bottom of the sheet pile,at four different depths, a square mesh of piezometers was constructed and installed at a distance of 10 × 10 cm with 44 piezometers. The diameter of the piezometers was 5 mm and they all were mounted on a special panel to read the water head under the dike. By choosing appropriate meshing, the piezometric pressures were extracted from the FLAC numerical model exactly at the points recorded in the laboratory model (Fig. 6).

The experiments were carried out for sheet pile depths of 0.1, 0.2, 0.3 and 0.4 m and five blanket lengths of 0, 1, 2, 3 and 3.5 m.

Fig. 6 Schematic of the board piezometers

3 Results and discussion

At the first step, the fine sand of foundation was saturated and as it can be in Fig. 7, water flows around the sheet pile and moves upward at downstream side of a sheet pile with 100 mm depth. Upward seepage flow at downstream of sheet pile, This was modified in this way. Due to seepage, effective stress at any point in depth z from the soil surface is reduced to $p = iz\gamma_w$ value, where i is hydraulic gradient, z is distance from surface and γ_w is water density. If the effective stress is reduced to zero, the soil particles become unstable and boiling will occur. In such condition, the hydraulic gradient is called critical hydraulic slope i_{cr} and is equal to 1 for most soils.

Figures 8, 9, and 10 indicate the results of the numerical model in comparison with experimental data. As it can be seen, the numerical model shows the values which are less than the experimental ones. The lower values of numerical model estimations may cause underestimation of the expected uplift pressure. Such underestimation may mislead the design engineer and jeopardize the safety of the structure.

In next step, the effects of foundation soil parameters such as internal friction angle ϕ, dilation angle φ and soil density were evaluated. To solve the problem of underestimation of the uplift pressure in prediction of the boiling potential, the internal friction and dilation angles can be chosen in a way that the soil critical condition is considered.

Based on the present study, increasing the friction and dilation angles, the possibility for boiling would reduce to occur. It is worthy of mentioning that dilation angle range for sand varies between 0° and 15° (FLAC-2D 2000). By selecting the internal friction angel $\varphi = 0$, the numerical model predicts the position of boiling properly for the case of L_B= 0, 1 m.

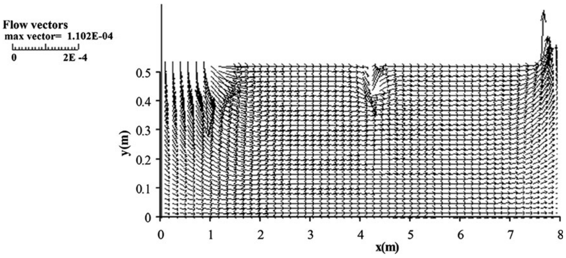


Fig. 7 Flow simulation around sheet pile installed at soil foundation

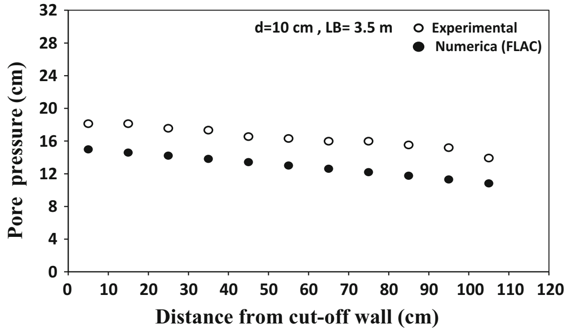


Fig. 8 Reduction of peizometric pressure for d = 10 cm, L_B = 3.5 m

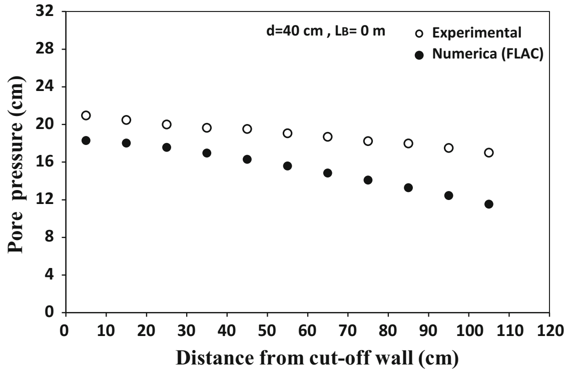


Fig. 9 Reduction of peizometric pressure for d = 40 cm, L_B = 0

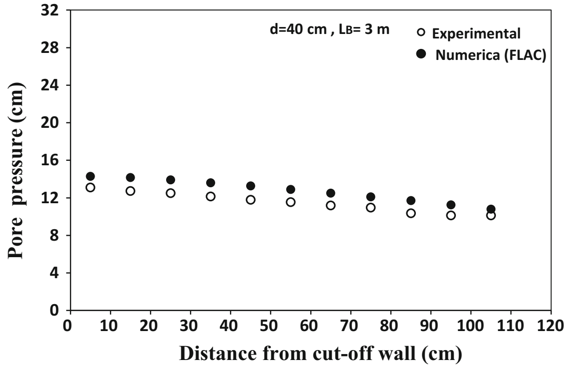


Fig. 10 Reduction of peizometric pressure for d = 40 cm, L_B = 3 m

Theoretically, the model is balanced when the resultant of applied forces in each node is zero. However, generally, the resultant of balanced forces in the model would be never zero in numerical analysis. When the unbalanced force with a known ratio is adequately small against loading condition, the balance condition is acceptable and satisfactory. When boiling occurs, the system will be practically unbalanced and the maximum unbalanced force is not close to zero that in Fig. 11, the x-axis represents the numbers of step and the y-axis of the unbalanced force.

Figure 12 shows strain rate at sheet pile depth of 100, 200, 300 and 400 mm, without blanket and sheet pile depth of 100 mm with 1 m blanket.

It is observed strain rate is maximum at downstream part. Figure 13 shows the near zero effective stress at downstream part. Theoretically, according to soil mechanic principles, sand boiling occurs when the pore water pressure increases until it is equal to the particles weight; in such a case, the effective stress is zero, sand particles would float and particle replacements occurs, due to loss of effective stress. Figure 14 shows one critical point at downstream part of dike in which effective stress is positive fluctuate around zero.

According to Fig. 15, when boiling occurs, the soil shear stress at downstream is reduced considerably and failure would happen. Figure 16 shows the grid transformations at downstream, causing upheaving as depicted in Fig. 17.

Comparing the results of the numerical model and experimental observations from the mechanical point of view in FLAC-2D with shows that there is a good agreement between them.

In other words, FLAC-2D has modeled boiling and predicted its behavior properly. A suitable option to prevent boiling is employing an impermeable blanket at downstream in order to increase the seepage path. Blanket lengths of 1, 2, 3 and 3.5 m have also been considered in the experimental model to control

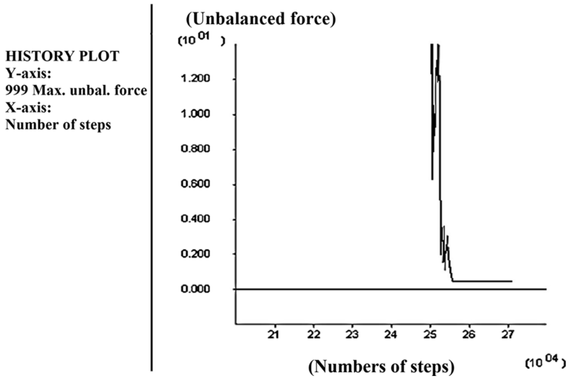


Fig. 11 Maximum unbalanced force diagram at boiling

Fig. 12 Strain rate at downstream of sheet pile at exit prism

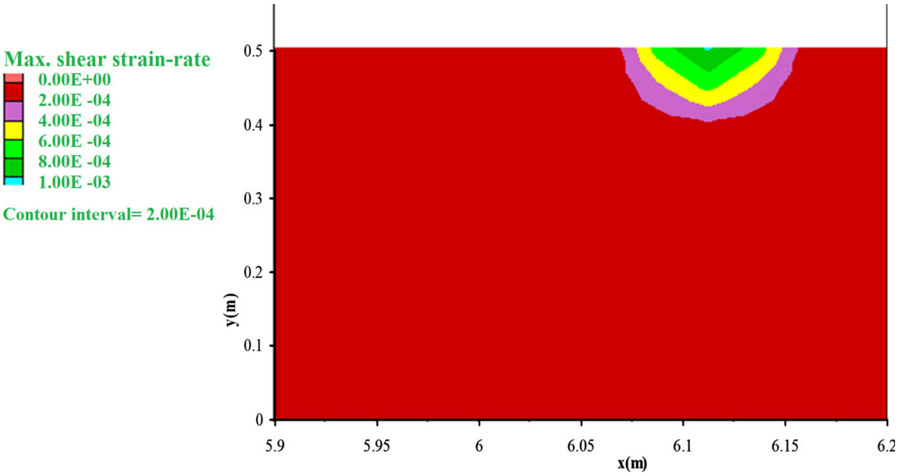
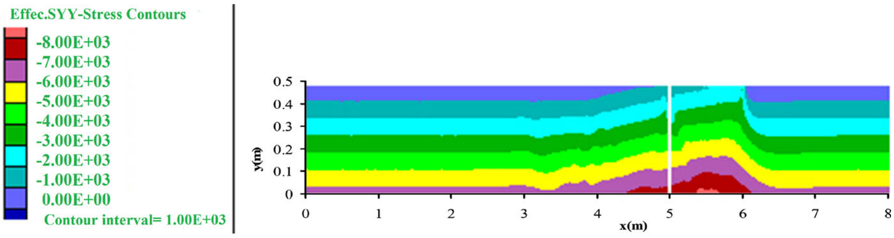


Fig. 13 Effective stress at downstream of sheet pile



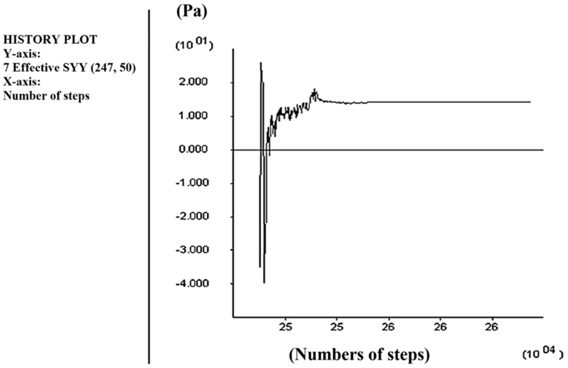


Fig. 14 Critical point effective stress at downstream of sheet pile

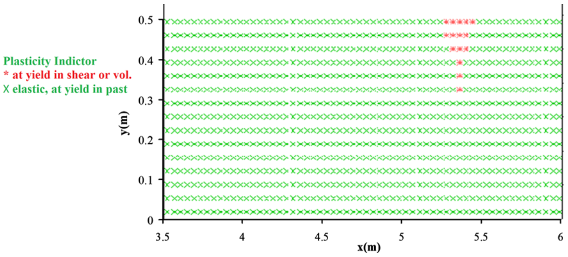


Fig. 15 Reduction of soil shear stress at downstream of sheet pile

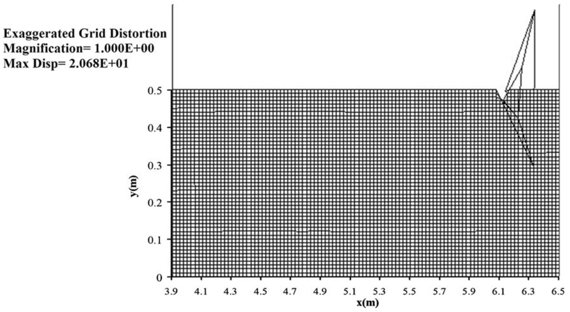


Fig. 16 Grid transformations at downstream at failure during boiling

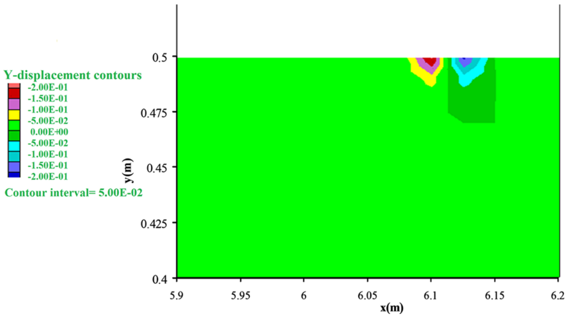


Fig. 17 Heaving at downstream while boiling

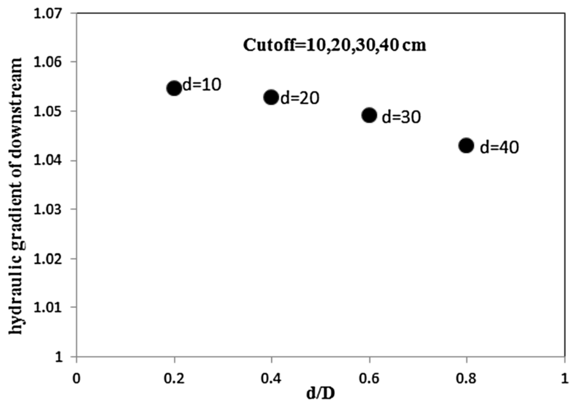


Fig. 18 Hydraulic gradient at downstream caused by increasing depth of sheet pile

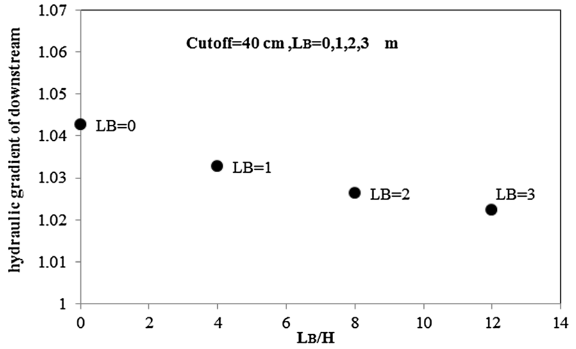


Fig. 19 Hydraulic gradient at downstream due to increasing length of blanket at d = 40 cm

boiling. According to Figs. 18 and 19 it is observed that by increasing depth of sheet pile or blanket length, hydraulic gradient would decrease and prevent boiling.

Table 4 presents the results for highly sensitive sandy soil, which shows full agreement between experimental and numerical modeling.

The results of FLAC-2D well predicted soil behavior and flow characteristics in alluvial foundation of coastal dikes. Stress–strain modeling of foundation is a complex modeling which it done in this paper. Hydraulic gradient and pore-pressure results of the numerical model have been verified with laboratory data of the flume. Alluvial foundation is the most critical material to occur boiling and heave instabilities which was predicted correctly by 2D modeling in this research.

Table 4 Comparison of results of numerical and experimental modeling for prediction of boiling phenomenon

Sheet pile depth (cm)		10	20	30	40
	Blanket (m)	Occurrence of boiling			
Experimental	0	+	+	+	−
Numerical		+	+	+	+
Experimental	1	+	−	−	−
Numerical		+	−	−	−
Experimental	2	−	−	−	−
Numerical		−	−	−	−
Experimental	3	−	−	−	−
Numerical		−	−	−	−
Experimental	3.5	−	−	−	−
Numerical		−	−	−	−

Boiling (+), not boiling (−)

4 Conclusions

Based on the results of the present study, the following conclusions can be made:

- By installation of seepage control measures such as sheet pile and/or blanket, it is therefore possible to decrease the hydraulic gradient of seepage flow and prevent boiling. The results of the paper indicated that FLAC-2D can perform numerical analysis seepage and boiling problems for sandy soil foundations.
- Based on Terzaghi equations, the safety factor against boiling is solely affected by porosity and particle density G_S. However, the numerical simulation indicates that dilation and internal friction angles also influence the safety factor and stability. It means that numerical analysis done by FLAC-2D is more reliable that empirical criterions.
- Solely considering the results from pore water pressure that creates uplift force, may cause underestimation of uplift force. For studying the boiling in planning, the soil behavior must be analyzed at the most critically geotechnical condition.
- While the elevation is in the form of a rectangular prism, its width is less than that the obtained prism by the Terzaghi method.
- It is purely based on the results of the piezometric pressure that produces uplift force. It may be

underestimating the uplift force. In order to study the boiling phenomenon in design, this numerical model in FLAC can analyze the soil's behavior in the most critical condition geotechnically.

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